ELLIOTT WAVES, FIBONACCI AND STATISTICS

by Robert R. Prechter, Jr.

"Retracements come in all sizes." –Frost and Prechter, *Elliott Wave Principle* (1998/2005), p.135

"[T]here is no significant difference between the frequencies with which price and time ratios occur in cycles in the Dow…." – Batchelor and Ramyar, "Magic Numbers in the Dow" (2005), p.18

A recent academic paper (Batchelor and Ramyar, 2005) investigated the frequency of price and time ratios attending adjacent movements in the DJIA ("retracements") as well as same-direction movements separated by an intervening movement ("projections"). Some comments from a practitioner may prove constructive.

The study is valuable in demonstrating that price-filtered movements in the stock market do not generally relate by a Fibonacci multiple either to price retracements or to projections. It supports an observation dating from the first edition of *Elliott Wave Principle* (Frost and Prechter) in 1978:

> In discerning the working of the Golden Ratio in the five up and three down movement of the stock market cycle, one might anticipate that on completion of any bull phase, the ensuing correction would be three-fifths of the previous rise in both time and amplitude. Such simplicity is seldom seen. (1978/2005, p.133)

The 1998 edition expanded upon this point:

> Retracements come in all sizes. Occasionally, a correction retraces a Fibonacci percentage of the preceding wave. [But these] ratios…are merely tendencies. Unfortunately, that is where most analysts place an inordinate focus because measuring retracements is easy. (1998/2005, p.135)

We stressed this point in reaction to an increasing tendency among some modelers and writers to ignore the specific observations within the Wave Principle and substitute a false claim that price-defined market movements in general are commonly related by Fibonacci percentages. Batchelor and Ramyar have performed a service in debunking this widespread, unsubstantiated belief. Their result agrees with our empirical observation, as quoted above.

In passing, Batchelor and Ramyar should be applauded for their observation, "There seems to be no logic for the ratios used by Gann…."(p.9) My own study (unpublished) of Gann's methods likewise found only numerology behind them. Further to their credit, the authors displayed an unbiased stance in citing Park and Irwin's 2004 review of 92 studies, from which Park and Irwin concluded, "…technical trading may be profitable in the long run even if technical trading strategies...are based on...'popular models' and not on information [exogenous to the market]." Batchelor and Ramyar thereby noted, "not all of technical analysis can be dismissed prima facie."

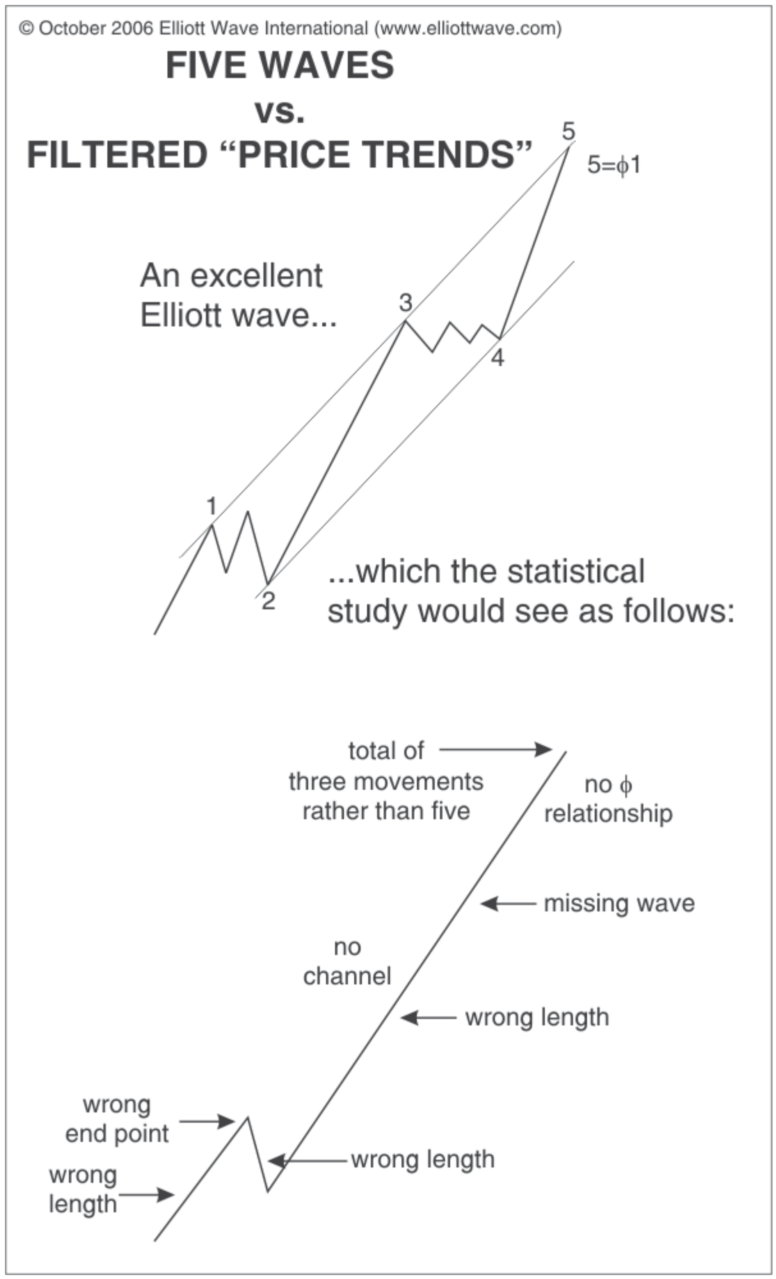
Not Applicable to Elliott Waves

Unfortunately, the authors also imputed to R.N. Elliott a generalization about the Wave Principle that he did not make. Elliott did note repeatedly that the "number of waves" in his model conforms to the Fibonacci sequence. But Batchelor and Ramyar asserted, "Elliott (1940) further claimed that the ratios of price and time retracements and projections in successive waves were likely to conform to Fibonacci ratios."(p.8) This statement is inaccurate. Through two books, a dozen articles and at least 60 periodicals, Elliott made no such claim.

Elliott's 1940 essay (1940/2005, pp.197-199) and a subsequent chapter in *Nature's Law* (1948/2005, pp.254-258) pointed out a single example of a period when four successive distances within "a triangular outline" are related in price approximately by the same Fibonacci ratio. Moreover, the first distance Elliott cited is not that of a "price trend" as defined by Batchelor and Ramyar's study but a net distance of three trends, leaving just two ratios that the method used in the study would discern. In a 1944 essay (1944/2005, pp.213-218) and a related subsequent chapter of *Nature's Law*, Elliott cited two instances in which a set of *multiple* waves is related by Fibonacci to a single wave in the same manner, a relationship the study is not designed to discern. In 1945, Elliott (1945/2005, p.133) used the Fibonacci ratio once (unsuccessfully) to support a market call, but again the relationship involved multiple waves, not single price trends. He never cited any other percentage retracement, never used the Fibonacci ratio for forecasting, never generalized about projections, and never generalized about retracements, not even in the narrow case of triangles.

Batchelor and Ramyar also investigate Fibonacci time relationships among price trends. But Elliott, who was perhaps overly intrigued with durations that last a Fibonacci number of time units, nevertheless said in 1941, "The time element as an independent device, however, continues to be baffling when attempts are made to apply any known rule of sequence to trend duration."(1941/2005, p.89) In other words, there is no time rule with respect to Elliott waves.

To conclude, the study does provide a service in debunking a certain widespread claim made by software designers and technicians who are not Elliott wave theorists and not even primarily Elliott wave analysts (the paper cites seven of them). But the study does not challenge the validity of any aspect of the Wave Principle; rather, as shown in the opening quotations and the following discussion, it supports wave theorists' observations.



Elliott Waves vs. "Price Trends"

The primary reason that the study does not pertain to Elliott waves is that Elliott waves differ from what the authors call "price trends," which are determined by a percentage price filter. Years ago technician Arthur Merrill (*Filtered Waves*, 1977) attempted to form conclusions about Elliott waves by using such a filter. He was unsuccessful because waves are a function of form, not price alone. Elliott wave forms involve both price and time. The accompanying figure shows a classic Elliott wave. In many cases, a price filter, which ignores time and form, would observe the latter part of wave 2 but none of wave 4. Instead of five waves, such a filter would discern three "price trends," ignoring two waves.

Elliott waves are defined as beginning and/or ending at certain points that are quite often different from the high and low prices within them. Such forms include two of the three types of corrective waves (flat and triangle) and "truncations" (in which ending prices do not reach a new price extreme) of the other three type of waves. So this fact either does or can pertain to all five wave types. Although truncations are rare, flats and triangles are common. Wave 2 in the accompanying figure is a flat, a single wave, yet its start is different from its price high. Wave 4 is a triangle, a single wave, yet its end is different from its price low. When determining ratios between Elliott waves, the start and end values are the defining points, not the intra-pattern extremes. Thus using a price filter to measure market movements ignores Elliott waves in two ways.

Application of Fibonacci Relationships by Elliotticians

Even within a proper Elliott-wave context, Elliott's successors have never formulated or applied any general rule about retracements or projections nor behaved as if one were true, and two of them (see quotes above) specifically denounced the idea. Charles Collins and Richard Russell did not use Fibonacci ratios at all. Hamilton Bolton cited a Fibonacci multiple just once in his career (1960/1994, pp.164, 279), A.J. Frost twice (1968 and 1970; 1998, pp.139, 164). (All three were forecasts, and two were successful.) Even among these few instances, none of them is based upon either a retracement or a projection of single alternate waves, which are the only wave relationships that the study's approach even occasionally would recognize. More telling, *Beautiful Pictures* (Prechter, 2003) presents 90 graphs, most of them containing multiple examples of price or time relationships in the DJIA, and none of them addresses a Fibonacci relationship between adjacent waves, and almost none of them addresses a "projection" between single alternate waves. Few of them even address relationships between "price trends" as defined in the study. Indeed, the book's Chapter 13, "Testing for Data Fitting," makes a case that when one ignores actual Elliott waves in favor of other price lengths, Fibonacci multiples between them almost never appear.

Chapter 4 of *Elliott Wave Principle* offers 14 idealized diagrams of situations where the authors discern Fibonacci relationships occurring more often than chance would allow. The methodology used in the Batchelor and Ramyar study fails to incorporate 10 of them. This is not a drawback to the authors' statistical method or the conclusion one may draw from it. It is, however, another indication that their method does not address actual claims by wave theorists.

The same book cites one real-life example of a Fibonacci retracement in the stock market. Even this one does not incorporate "price trends" such as the study uses but instead is based upon two Elliott waves whose turning points differ from the market's price extremes. The example conforms to the observation in *Elliott Wave Principle* that Fibonacci retracements occur "occasionally" among *particular* pairs of waves, in this case 1 and 2. While even this statement may ultimately prove false, the paper's statistics do not address its validity.

Batchelor and Ramyar conclude, "So in Figure 3, we might expect the retracement ratio of the price range between turning points 2 and 3 to be a Fibonacci ratio multiple of the range between points 1 and 2."(p.8) Aside from the general inapplicability to the Wave Principle already established, this statement contradicts the fact that no Elliott theorist has proposed any theoretically ideal price relationship or set of price relationships between those particular waves (2 and 3). To my knowledge, there are none. *Elliott Wave Principle* and *Beautiful Pictures* specifically decline to suggest any reliable relationship between these two waves. Randomness found here would simply support Elliott theorists' observations.

As parts of this discussion indicate, two Elliott theorists do claim to have observed a better than random chance of finding Fibonacci price relationships between certain specific types of waves and wave groupings. Based on Elliott's and the authors' own observations, *Elliott Wave Principle* asserts, "Far more precise and reliable, however, are relationships between [certain] alternate waves, or lengths unfolding in the same direction...." (1998/2005, p.135) The guidelines offered for impulse waves all involve either groupings of waves or individual waves separated by three waves, not single successive waves or price trends. The Fibonacci guidelines offered for corrections do involve immediately successive single waves, but in more than half of these instances price extremes and wave endings either can or do differ, distinguishing these waves from price trends. While testing all such claims remains to be accomplished, and while testing may prove that even these claims are invalid, it would appear that Elliott's successors' inability to find any other reliable wave relationships suggests that (1) Elliott theorists have been able to make distinctions between situations in which a Fibonacci relationship is probable relative to all others, and (2) they would agree that testing relationships outside these few narrow claims would likely yield a random result.

If anything, then, the conclusion of this study supports the observations of Elliott theorists, who say (directly or indirectly) three things that would anticipate the overall statistical result obtained: (1) There are no Fibonacci-based retracement or projection rules relating generally to all waves or "price trends"; (2) only certain combinations of waves seem to display Fibonacci relationships more often than chance, whereas all others do not display them; (3) investigating which market movements, if any, tend to have a Fibonacci relationship would require using actual Elliott waves, not price trends as defined by Batchelor and Ramyar, and the use of such price trends would tend to give a result close to random.

Statistics and Significance

Wave theorists propose only a limited number of circumstances in which a Fibonacci relationship is likely, and even then the probability for such a relationship is quite less than 100 percent. If these wave theorists are correct, and if Batchelor and Ramyar's study is set up to discern a small portion of such conditions, then shouldn't their statistics reflect a slight degree of non-randomness regarding the appearance of Fibonacci ratios in stock market movements? Perhaps they do.

Deepak Goel of the Socionomics Institute finds in Batchelor and Ramyar's statistics a different message from what reviewers to date have concluded. He observes that although the study's Fibonacci table does show that occurrence rates for individual ratios are not always significant, it also shows that higher-than-expected occurrence rates are numerous to a degree that is highly statistically significant. In other words, the reported results do indicate a degree of non-randomness for Fibonacci ratios in retracements and projections. Please see Goel's analysis at www.socionomics.net/FiboStats.

Anyone who works with Elliott waves would be quite at home with this interpretation of the results. Lumping in a few legitimate wave relationships with the majority of "retracements" and "projections" that wave theorists would deem inapplicable would produce just the kind of slightly-better-than-random results that the authors report. The relevant statistics, therefore, may support the observation in *Elliott Wave Principle* that Fibonacci relationships are to be expected in certain specific situations.

Testing

In speeches, I often remark that the herding impulse is a blunt instrument developed through evolution to cope with the challenges faced by more primitive creatures in a more primitive environment. But the bluntness of some statistical studies hardly seems of a different order. The authors note, "[T]he proposition that stock prices retrace to such levels is unusual among technical trading rules, in the sense that it can be clearly formulated in numeric terms, and is potentially testable."(p.10) This statement is correct for a sad reason. As noted in *Elliott Wave Principle*, some writers have asserted this proposition about the market not because it is valid but "because measuring retracements is easy." I sympathize with statisticians who want to test simple trading rules, but the Wave Principle model, like the market itself, is not simple. The Wave Principle is fairly well defined, though, and surely testable with the right definitions and the right tools. Though our resources are limited, my colleagues and I would be happy to help in any way we can. In the meantime, it is heartening to see statistical studies whose results appear to be consistent with our empirical observations.

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LINKS AND SOURCES

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[For the books by Elliott wave theorists, see www.elliottwave.com/books.]